# **Promises and Pitfalls of Threshold-based Auto-labeling**

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#### What & Why auto-labeling?

Data labeling problem

Wide adoption of auto-labeling

Finding the auto-labeling region

## Roadmap

#### How does it work?

Workflow of TBAL

#### Analysis & Results

Conditions when TBAL works.

Comparison with Active Learning, Selective Classification



## We need labeled data and often a lot of it!

# Diagnosing a novel disease using brain scans



### Fine-tuning Foundation models or Aligning LLMs



## Data Labeling costs a lot of time and money

### Crowdsourcing is widely used to get labels

**Wisdom of Crowd** 





Takes a lot of time and money to get labels.

IMAGENET Deng et. Al. 2009

#### Took multiple years and a lot of human effort



A screenshot of the ImageNet database online



# How do we get accurately labeled data, while spending less time and money?



### Automatically label datasets with minimal human feedback Human-labeled



#### Get labels for "minimal" points from human



Human Labeled data



#### Auto-label using the model





Train a model on these labeled points and









00

# **Auto-Labeling Errors and Their Impact**



**Unlabeled Data** 

Human-labeled Auto-labeled 00

Labeled Data

# **Auto-Labeling Errors and Their Impact**



### **1.** The output dataset may have labeling errors

### 2. The impact of errors in datasets is more severe

a) Multiple downstream applications b) Longer shelf-life than models.



## Auto-labeling systems are widely used

#### **Auto-labeling Platforms**







### Despite wide adoption, our understanding of auto-labeling systems is limited!

## Despite wide adoption, our **understanding of auto-labeling systems is limited!**

# To address this gap we develop a theoretical understanding of auto-labeling systems.



## Auto-labeling systems are widely used





**Even in high risk applications** 

health care, telecom, recruiting...

So we need to understand them.





#### What & Why auto-labeling?

Data labeling problem

Adoption of auto-labeling

#### How does it work?

Workflow of TBAL

Finding the auto-labeling region

## Roadmap

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# **Quality and Quantity of Auto-labeled Data**

Auto-labeled 00

0

0

maximize this



Number of Nunlabeled points

00 0 Set of auto-labeled points

Ο

 $N_a$ Number of auto-labeled points

## Quantity **Auto-labeling Coverage** Good Stuff

N



Unknown Auto-labeled 00 **True Decision Boundary** X Labeling mistake X Ο X 0

 $M_a$  Number of labeling mistakes

### Quality **Auto-labeling Error**

$$\widehat{\mathcal{E}} = \frac{M_a}{N_a}$$

**Bad Stuff** minimize this



### Threshold-based Auto-labeling Workflow(TBAL)





Pretend we are LLMs and

# Let's think step by step with an example



### **Threshold-based Auto-labeling Workflow(TBAL)**

#### Input

### **Unlabeled Data** i.i.d from space X



 $\epsilon_a$ 



# $\begin{array}{l} \textbf{Model Class} \\ \mathcal{H}: \mathcal{X} \mapsto \mathcal{Y} \\ h(\mathbf{x}; \mathbf{w}) = \operatorname{sign}(\mathbf{w}^T \mathbf{x}) \end{array} \end{array}$

#### **Confidence Function**

$$g: \mathcal{X} \mapsto T \subseteq \mathbb{R}^+$$
$$g(\mathbf{x}; \mathbf{w}) = |\mathbf{w}^T \mathbf{x}|$$



#### **Unlabeled Data**

i.i.d from space  $\mathcal{X}$ 



### **Learning** $f^*$ is NOT the goal.

#### Input



**Unlabeled Data** 

**Auto-labeling** error tolerance

 $\epsilon_a$ 

0.0 00 0 Ο 0 О O

### **Expected Output**



Human-labeled Auto-labeled 00 X Labeling mistake

### Auto-labeling Error

$$\widehat{\mathcal{E}} = \frac{M_a}{N_a} = \frac{\# \times}{\# \circ + \# \circ} \leq$$

Coverage

$$\hat{\mathcal{P}} = \frac{N_a}{N} = \frac{\texttt{\#O} + \texttt{\#O}}{\texttt{\#O}}$$





### **TBAL Workflow : Bootstrap (Step 0) Pick a Model class and Confidence function**

$$egin{aligned} \mathcal{H} : \mathcal{X} \mapsto \mathcal{Y} \ \mathcal{X} = \{\mathbf{x} \in \mathbb{R}^2 : ||\mathbf{x}||_2 \leq 1\} \ \mathcal{Y} = \{-1, +1\} \end{aligned}$$

### **Confidence/Scoring Function**

$$g: \mathcal{X} \mapsto T \subseteq \mathbb{R}^+$$
$$T = [0, 1]$$



### **Model/Hypothesis Class**

### **Linear Classifiers** $\mathcal{W} = \{ \mathbf{w} \in \mathbb{R}^2 : ||\mathbf{w}||_2 \le 1 \}$ $h(\mathbf{x}; \mathbf{w}) = \operatorname{sign}(\mathbf{w}^T \mathbf{x})$ $\notin \mathcal{H}$



#### **Linear Confidence** Function $g(\mathbf{x}; \mathbf{w}) =$ $1 + e^{-|\mathbf{w}^T \mathbf{x}|}$

 $\equiv |\mathbf{w}^T \mathbf{x}|$ 





### **TBAL Workflow : Bootstrap (Step 0)** Pick a Model class and Confidence function

### **Model/Hypothesis Class**

$$\mathcal{H}: \mathcal{X} \mapsto \mathcal{Y}$$
  
 $\mathcal{X} = \{\mathbf{x} \in \mathbb{R}^2 : ||\mathbf{x}||_2 \leq 1\}$   
 $\mathcal{Y} = \{-1, +1\}$ 

### **Confidence/Scoring Function**

$$g: \mathcal{X} \mapsto T \subseteq \mathbb{R}^+$$
$$T = [0, 1]$$





### **TBAL Workflow : Bootstrap (Step 0)** Get some labeled data for training and validation



### **Training Set**

 $D_{train} = \{(\mathbf{x}_i, y_i) : i \in I_{train}\}$ Start small and gradually add more **Unlabeled Set** 

#### **Validation Set**

 $D_{val} = \{ (\mathbf{x}_i, y_i) : i \in I_{val} \}$ 

Get "sufficiently" large amount of it.



### Threshold-based Auto-labeling Workflow(TBAL)

#### Input

### **Unlabeled Data** i.i.d from space X



 $\epsilon_a$ 





### **TBAL Workflow : Step 1 Model training**



**Training Set** 



$$\hat{h} = \texttt{EmpiricalRis}$$
  
 $\hat{h} = \operatorname*{arg\,min}_{h \in \mathcal{H}} rac{1}{|D_{train}|}$ 

In practice, usually some surrogate loss is minimized



### **Threshold-based Auto-labeling Workflow(TBAL)**



![](_page_24_Figure_2.jpeg)

**Confidence Function**  $g: \mathcal{X} \mapsto T \subseteq \mathbb{R}^+$  $g(\mathbf{x}; \mathbf{w}) = |\mathbf{w}^T \mathbf{x}|$ 

![](_page_24_Figure_4.jpeg)

### Learn a model w using training set Empirical Risk Minimizer from $\mathcal{H}$

![](_page_24_Figure_8.jpeg)

### Find the Auto-labeling region

### Idea 1: Auto-label everywhere.

![](_page_25_Figure_3.jpeg)

![](_page_25_Picture_4.jpeg)

### Find the Auto-labeling region

### Idea 1: Auto-label everywhere.

![](_page_26_Figure_3.jpeg)

![](_page_26_Picture_4.jpeg)

![](_page_26_Figure_5.jpeg)

### **Find the Auto-labeling region**

### Idea 1: Auto-label everywhere.

![](_page_27_Figure_3.jpeg)

![](_page_27_Picture_4.jpeg)

![](_page_27_Figure_5.jpeg)

**Could lead to high auto-labeling errors!** 

![](_page_27_Picture_9.jpeg)

![](_page_28_Picture_0.jpeg)

Panda's strategy does not work, he goes to Master Shifu for advice.

![](_page_28_Picture_2.jpeg)

### Idea 2: Auto-label where the model is accurate (or trustworthy?)

![](_page_29_Figure_2.jpeg)

#### Learned Model

### How to find the yellow and green regions?

![](_page_29_Figure_5.jpeg)

![](_page_29_Picture_7.jpeg)

![](_page_30_Figure_2.jpeg)

### Use the validation data to find the region where the classifier can be trusted

Trust Here

![](_page_31_Figure_1.jpeg)

Predictions sorted by confidence scores

![](_page_31_Figure_3.jpeg)

$$g(\mathbf{x}; \hat{\mathbf{w}}) = |\hat{\mathbf{w}}^T \mathbf{x}|$$

Regions defined by the confidence function

 $A_v(\hat{\mathbf{w}}, t, y) = \{ \mathbf{x} \in X_v : g(\mathbf{x}; \hat{\mathbf{w}}) \ge t, \hat{h}(\mathbf{x}, \hat{\mathbf{w}}) = y \}$ 

Auto-labeling Error estimation in these regions

$$\hat{E}_{v}(\hat{\mathbf{w}}|t,y) = \frac{1}{|A_{v}(\hat{\mathbf{w}},t,y)|} \sum_{\mathbf{x}\in A_{v}(\hat{\mathbf{w}},t,y)} \mathbb{1}\{\hat{h}(\mathbf{x};\hat{\mathbf{w}})\neq f^{\star}(\mathbf{x})\}$$

$$\mathbf{A} = \frac{\# \mathbf{X}}{\# \mathbf{V} + \# \mathbf{X}}$$

![](_page_31_Picture_11.jpeg)

![](_page_31_Picture_12.jpeg)

![](_page_32_Figure_0.jpeg)

![](_page_33_Figure_1.jpeg)

![](_page_33_Figure_2.jpeg)

 $g(\mathbf{x}; \hat{\mathbf{w}}) = |\hat{\mathbf{w}}^T \mathbf{x}|$ 

 $A_v(\hat{\mathbf{w}}, t, y) = \{ \mathbf{x} \in X_v : g(\mathbf{x}; \hat{\mathbf{w}}) \ge t, \hat{h}(\mathbf{x}, \hat{\mathbf{w}}) = y \}$ 

Predictions sorted by confidence scores

![](_page_33_Figure_6.jpeg)

![](_page_33_Figure_7.jpeg)

![](_page_33_Figure_9.jpeg)

![](_page_34_Figure_1.jpeg)

$$g(\mathbf{x}; \hat{\mathbf{w}}) = |\hat{\mathbf{w}}^T \mathbf{x}|$$

![](_page_35_Figure_1.jpeg)

$$g(\mathbf{x}; \hat{\mathbf{w}}) = |\hat{\mathbf{w}}^T \mathbf{x}|$$

 $A_v(\hat{\mathbf{w}}, t, y) = \{ \mathbf{x} \in X_v : g(\mathbf{x}; \hat{\mathbf{w}}) \ge t, \hat{h}(\mathbf{x}, \hat{\mathbf{w}}) = y \}$ 

![](_page_35_Figure_5.jpeg)

![](_page_36_Figure_1.jpeg)

 $g(\mathbf{x}; \hat{\mathbf{w}}) = |\hat{\mathbf{w}}^T \mathbf{x}|$ 

#### Cannot find a threshold on this side.

![](_page_37_Figure_1.jpeg)

$$g(\mathbf{x}; \hat{\mathbf{w}}) = |\hat{\mathbf{w}}^T \mathbf{x}|$$

![](_page_37_Figure_4.jpeg)

![](_page_37_Figure_5.jpeg)

![](_page_38_Figure_1.jpeg)

$$g(\mathbf{x}; \hat{\mathbf{w}}) = |\hat{\mathbf{w}}^T \mathbf{x}|$$

$$A_v(\hat{\mathbf{w}}, t, y) = \{\mathbf{x} \in X_v : g(\mathbf{x}; \hat{\mathbf{w}}) \ge t, \hat{h}(\mathbf{x}, \hat{\mathbf{w}}) =$$

![](_page_38_Figure_4.jpeg)

![](_page_38_Picture_6.jpeg)

Find the Auto-labeling region

![](_page_39_Figure_2.jpeg)

$$g(\mathbf{x}; \hat{\mathbf{w}}) = |\hat{\mathbf{w}}^T \mathbf{x}|$$

### Threshold-based Auto-labeling Workflow(TBAL)

![](_page_40_Figure_1.jpeg)

![](_page_40_Figure_2.jpeg)

## **TBAL Workflow: Step 3** Auto-label points in the identified region

We found a threshold that has error <  $\epsilon_a$ 

![](_page_41_Picture_2.jpeg)

![](_page_41_Figure_3.jpeg)

![](_page_41_Figure_4.jpeg)

Auto-labeled

![](_page_41_Picture_8.jpeg)

### Threshold-based Auto-labeling Workflow(TBAL)

![](_page_42_Figure_1.jpeg)

### **TBAL Workflow: Step 4** Prepare for the next round

Remove auto-labeled points from the pool.

![](_page_43_Picture_2.jpeg)

#### Remaining unlabeled data

Remove points from the validation set Falling in the auto-labeling region.

![](_page_43_Picture_5.jpeg)

Remaining validation data

### Threshold-based Auto-labeling Workflow(TBAL)

![](_page_44_Figure_1.jpeg)

![](_page_44_Figure_4.jpeg)

### Step 5: Query next batch of human-labeled data for training

Use some active querying strategy example: uncertainty sampling

![](_page_45_Picture_2.jpeg)

#### Next round's training data

![](_page_45_Figure_4.jpeg)

### If there is unlabeled data left

Go to Step 1

![](_page_45_Picture_8.jpeg)

### Intermediate Rounds Output

![](_page_46_Figure_1.jpeg)

![](_page_46_Picture_2.jpeg)

### Threshold-based Auto-labeling Workflow(TBAL)

![](_page_47_Figure_1.jpeg)

![](_page_47_Figure_3.jpeg)

00 X

Auto-labeled data in the end

![](_page_48_Picture_4.jpeg)

![](_page_48_Figure_5.jpeg)

# Final Output

Human-labeled Auto-labeled Labeling mistake

### Output Labeled Dataset

#### Error and Coverage

Auto-labeling Error < 1%

Coverage > 95%

![](_page_48_Picture_13.jpeg)

![](_page_49_Picture_0.jpeg)

#### What & Why auto-labeling?

Data labeling problem

Adoption of auto-labeling

![](_page_49_Figure_4.jpeg)

Finding the auto-labeling region

## Roadmap

#### How does it work?

Workflow of TBAL

#### Analysis & Results

#### Conditions when TBAL works.

Comparison with Active Learning, Selective Classification

![](_page_49_Picture_13.jpeg)

### **Theoretical Results**

### **Conditions on the validation data for accurate auto-labeling**

![](_page_50_Figure_2.jpeg)

### In the general setup: No assumptions on data distribution and function classes

### **Upper bound on excess auto-labeling error**

$$\mathcal{O}\left(\frac{1}{\sqrt{N_v}} + \mathfrak{R}_{N_v}(\mathcal{H}^{T,g})\right)$$

### Lower bound on number of validation samples to ensure auto-labeling error is below $\epsilon_a$

$$\Omega\!\left(\frac{1}{\epsilon_a^2}\right)$$

![](_page_50_Figure_8.jpeg)

![](_page_50_Figure_9.jpeg)

$$\mathcal{H}^{T,g} := \mathcal{H} \times T \quad (h,t) \in \mathcal{H}^{T,g}$$
$$(h,t)(\mathbf{x}) := \begin{cases} h(\mathbf{x}) & \text{if } g(h,\mathbf{x}) \ge \\ \text{abstain} & \text{o.w.} \end{cases}$$

Instantiate the upper bound for uniform distribution on unit-ball in  $\mathbb{R}^d$ with homogeneous linear separators

![](_page_50_Picture_14.jpeg)

## **Proof Sketch**

# With Finite Samples $A_v(h,t) = \{ \mathbf{x} \in X_v : g(\mathbf{x};h) \ge t \}$

![](_page_51_Figure_2.jpeg)

![](_page_51_Figure_3.jpeg)

w.p. 
$$1 - \delta$$
  
 $\mathcal{E}(h|t) \le \widehat{\mathcal{E}}_v(h|t) + \psi(N)$ 

#### Population Level

$$\mathcal{A}(h,t) = \{\mathbf{x} \in \mathcal{X} : g(\mathbf{x};h) \ge t\}$$

 $\mathcal{E}(h|t) = \mathbb{E}_{x|\mathcal{A}(h,t)}[\mathbb{1}\{h(\mathbf{x}) \neq f^{\star}(\mathbf{x})\}]$ 

![](_page_51_Figure_8.jpeg)

#### Want this

### $V_v, \delta, \mathcal{H}, g, T) \quad \forall h \in \mathcal{H}, \forall t \in T$

### **Proof Sketch**

![](_page_52_Figure_1.jpeg)

![](_page_52_Figure_2.jpeg)

$$\begin{aligned} \mathcal{E}(h,t) &= \mathbb{E}_{\mathbf{x}}[\mathbbm{1}\{h(\mathbf{x}) \neq f^{\star}(\mathbf{x})\} \wedge \mathbbm{1}\{g(\mathbf{x}) \geq t\}] \\ \mathbb{P}(h,t) &= \mathbb{E}_{\mathbf{x}}[\mathbbm{1}\{g(\mathbf{x}) \geq t\}] \qquad \mathcal{E}(h|t) = \frac{\mathcal{E}(h,t)}{\mathbb{P}(h,t)} \end{aligned}$$

$$\widehat{\mathcal{E}}_{v}(h,t) = \frac{1}{N_{v}} \sum_{\mathbf{x}_{i} \in X_{v}} \mathbb{1}\{h(\mathbf{x}_{i}) \neq f^{\star}(\mathbf{x}_{i})\} \land \mathbb{1}\{g(\mathbf{x}_{i}) \ge t\}$$
$$\widehat{P}_{v}(h,t) = \frac{1}{N_{v}} \sum_{\mathbf{x}_{i} \in X_{v}} \mathbb{1}\{g(\mathbf{x}_{i}) \ge t\} \qquad \widehat{\mathcal{E}}_{v}(h|t) = \frac{\widehat{\mathcal{E}}_{v}(h,t)}{\widehat{P}_{v}(h,t)}$$

#### Uniform convergence results

![](_page_52_Picture_8.jpeg)

![](_page_53_Picture_0.jpeg)

Experiments

## **Active Learning and Selective Classification**

### **Active Learning (AL)**

$$\operatorname{err}(h) = \mathbb{E}_{\mathbf{x}}[\mathbb{1}\{h(\mathbf{x}) \neq y\}]$$
$$h^* \in \operatorname{arg\,min}_{h \in \mathcal{H}} \mathbb{E}_{\mathbf{x}}[\mathbb{1}\{h(\mathbf{x}) \neq y\}]$$
$$\operatorname{err}(\hat{h}) - \operatorname{err}(h^*) \to 0$$

![](_page_54_Figure_3.jpeg)

Cohn et al. 1994;

Balcan, Dasgupta, Nowak, Zhu, Hanneke, Jamieson,

Chaudhury.... (Over the last 3 decades)

### **Selective Classification (SC)**

![](_page_54_Figure_9.jpeg)

El-Yaniv & Weiner, 2010; Cortes, Desalvo, Mohri 2016; Gelbhart & El-Yaniv 2019; Fisch, Jakkola et al. 2022;

#### A natural auto-labeling strategy (AL+SC): First learn the best classifier using Active Learning, then auto-label using selective classification.

# The methods work as expected on the circles example

#### Misspecified setting: Using incorrect model class, (in practice the correct class is not known)

![](_page_55_Figure_2.jpeg)

![](_page_55_Figure_3.jpeg)

![](_page_55_Picture_5.jpeg)

## We validate the results empirically

Vary the number of validation points

	N	Error (%)		Coverage (%)	
		TBAL	AL+SC	TBAL	AL+SC
	100	$3.10{\scriptstyle~\pm1.80}$	$0.68{\scriptstyle~\pm 0.81}$	$71.43 {\scriptstyle~\pm 8.86}$	$96.95{\scriptstyle~\pm1.01}$
	400	$1.65 \pm 0.65$	$0.32{\scriptstyle~\pm 0.15}$	93.27 ±2.50	96.91 ±0.99
	800	$1.08 \pm 0.47$	$0.24 \pm 0.16$	96.01 ±1.16	$96.31 \pm 1.36$
	1200	$0.78{\scriptstyle~\pm 0.27}$	$0.17 \pm 0.11$	$96.82{\scriptstyle~\pm 0.84}$	$95.96 \pm 1.40$
	1600	$0.65{\scriptstyle~\pm 0.20}$	0.13 ±0.08	$96.93 \pm 0.57$	$95.70{\scriptstyle~\pm1.38}$
	2000	$0.54{\scriptstyle~\pm 0.16}$	0.21 ±0.11	$97.23{\scriptstyle~\pm 0.42}$	96.36 ±1.13

Validation data

Increasing

#### **Unit ball (Synthetic)**

N.,	Error (%		
	TBAL	A	
200	$2.28{\scriptstyle~\pm 0.21}$	3.	
400	$1.29{\scriptstyle~\pm 0.10}$	1.	
600	$1.41 \pm 0.20$	1.	
800	$1.62 \pm 0.30$	2.	
1000	$1.64 \pm 0.23$	1.	

# Classes = 2  $\epsilon_a$  = 1% Max # training points = 500

![](_page_56_Figure_8.jpeg)

Fix the auto-labeling error tolerance and the max number of training points algorithm can use.

#### IMDB

#### Coverage (%) %) **TBAL** AL+SC AL+SC $68.24 \pm 6.20$ 57.77 ±13.09 $.11 \pm 0.86$ $63.81 \pm 4.86$ $63.06 \pm 10.70$ $.98 \pm 0.40$ .81 ±0.22 $62.92 \pm 9.20$ 69.64 ±3.98 67.45 ±3.72 $63.22 \pm 7.89$ $.04 \pm 0.35$ $.97 \pm 0.26 | 70.28 \pm 2.82$ $66.11 \pm 8.00$

#### **Tiny Imagenet**

N	Erro	Coverage		
	TBAL	AL+SC	TBAL	
2000	$0.0 \pm 0.0$	$0.0{\scriptstyle~\pm 0.0}$	$0.0 \pm 0.0$	
4000	$10.50{\scriptstyle~\pm 6.01}$	$7.37{\scriptstyle~\pm4.57}$	$0.47 \pm 0.05$	(
6000	$10.61 \pm 0.62$	7.71 ±1.03	$10.16 \pm 1.10$	Ζ
8000	$9.90{\scriptstyle~\pm 0.63}$	$6.80{\scriptstyle~\pm 0.77}$	$25.84 \pm 1.57$	1
10000	$8.97 \pm 0.36$	$6.87{\scriptstyle~\pm 0.48}$	$32.19{\scriptstyle~\pm1.34}$	2

# Classes = 2  $\epsilon_a$  = 5% Max # training points = 500

 $\epsilon_a$ = 10% # Classes = 200 Max # training points = 10000

#### As expected, we observe

high auto-labeling errors and high variance in coverage

less auto-labeling errors and less variance in coverage

![](_page_56_Figure_20.jpeg)

![](_page_56_Picture_21.jpeg)

![](_page_56_Picture_22.jpeg)

# **Summary and Takeaways**

![](_page_57_Figure_1.jpeg)

1. Auto labeling is a promising solution to obtain labeled data.

#### **Threshold-based Auto-labeling Workflow**

- 2. Our work develops a theoretical understanding of auto-labeling systems.
- 3. The promise Seemingly bad models can auto-label significant portion of data with good accuracy.
- 4. The pitfall Hidden downside is it may need large amount validation data to ensure good accuracy.

![](_page_57_Picture_11.jpeg)

![](_page_57_Picture_12.jpeg)

![](_page_58_Picture_0.jpeg)

#### **Checkout our paper and code! Come to our poster @ NeurIPS**

![](_page_58_Picture_2.jpeg)

Paper <a href="https://openreview.net/pdf?id=RUCFAKNDb2">https://openreview.net/pdf?id=RUCFAKNDb2</a> Code <a href="https://github.com/harit7/TBAL-NeurIPS-23">https://github.com/harit7/TBAL-NeurIPS-23</a>

Thank You

0 Hall B1 + B2 #1103 Wed 13 Dec 3 p.m. - 5 p.m. PST

#### **Contact us**

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